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FULL PAPER

Topological indices of the system of generalized prisms *via* M-Polynomial approach

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The characteristics of various networks can be distinguished with the help of topological indices. The purpose of this paper is to study the generalized prism network, which is very interesting for physics and engineering researchers. Regarding this network, we recovered some degree-based topological indices from the M-polynomial. We measured topological indices such as atom-bond connectivity, geometric arithmetic, K Banhatti, K hyper Banhatti, modified K Banhatti and harmonic K Banhatti by using M-polynomial approach.

KEYWORDS

Topological indices; M-polynomial; generalized prism net-work.

Introduction

Graph theory has grown into an available area of mathematical research with implementation in chemistry, operation researches, social sciences and computer science. Due to variety of research in graph theory, this field has become a large subject in mathematics.

Graph theory has vast applications in diverse fields of science and engineering. Topological indices are graph invariants and have large number of applications. Here, we did computation via M-polynomial approach. This technique was introduced in 2015 [29]. Later on, this approach has been adopted by some other scholars [2]. Several works have been done in this area using these techniques.

M-polynomial of many graphs have been introduced over last five years [26,33]. The main advantage of the M-polynomial is the large amount of information that it contains about degree-based graph invariants.

Topological indices are computed by definition. Various topological indices that are derived from graph theory can model the geometric structure of chemical compounds. The study of topological indices, based on distance in a graph, was effectively employed in chemistry by Weiner [38].

Newly established degree-based topological indices have been formulated via M-polynomial [1-25]. We can find various degree dependent topological invariants given in Table 1. In this paper, we calculated the topological indices of generalized prism $P\{n,m\}$.

Here, some degree-based topological indices via M-polynomial approach using the table given in [3] were computed.

Generalized prism network

The cycle graph C_n and path graph P_m are combined in the generalized prism structure.

Generalized prism graph *P{m,n}* is obtained by cartesian product of a cycle C_n with a path P_m. Then

$V(P_{\{n,m\}}) = \{x_i x_j : 1 \le i \le n, 1 \le j \le m\}$

and

$E(P_{\{n,m\}}) = \{(x_i y_j)(x_{i+1} y_j) : 1 \le i \le n-1, 1 \le j \le m\}$

Ò $\{(x_n y_j)(x_1 y_j) : 1 \le j \le m\}$

Ò $\{(x_i y_j)(x_i y_{j+1}) : 1 \le i \le n, 1 \le j \le m-1\}$.

The generalized prism *P{n,m}* has been studied [27,29,32,33,35,36].

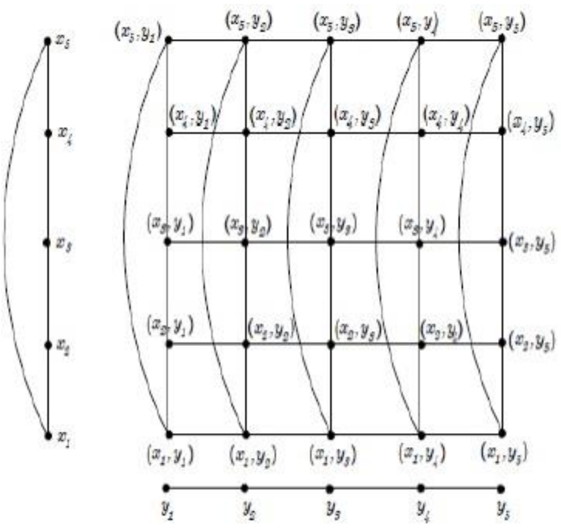

FIGURE 1 Generalized prism graph [32].

M-Polynomial of Generalized Prism Network

Generalized prism network is denoted by *P{n,m}* then M-polynomial of *P{n,m}* computed in [26] is represented by M(P{n,m},x,y) and it is

$M(P\{n,m\},x,y)=g(x,y)$

$=(2n)x^3y^3+(2n)x^3y^4+(2mn-5n)x^4y^4$

Topological Indices of Generalized Prism Network

This section deals with our results.

Theorem 4.1. *Let P{n,m} be the generalized prism and* $M(P\{n,m\},x,y)=g(x,y)$ $=(2n)x^3y^3+(2n)x^3y^4+(2mn-5n)x^4y^4$. *Then*

$$[1.]ABC[P_{\{n,m\}}] = (16+4\sqrt{15}-15\sqrt{6})\frac{n}{12}+\frac{\sqrt{6}}{2}mn$$

$$[2.]GA[P_{\{n,m\}}] = (\frac{-21+8\sqrt{3}}{7})n+2mn$$

$$[3.]B_1[P_{\{n,m\}}] = -38n+40mn$$

$$[4.]B_2[P_{\{n,m\}}] = -122n+96mn$$

$$[5.]HB_1[P_{\{n,m\}}] = -514n+400mn$$

$$[6.]HB_2[P_{\{n,m\}}] = -3934n+2304mn$$

$$[7.]^m B_1[P_{\{n,m\}}] = \frac{11}{252}n+\frac{2}{5}mn$$

$$[8.]^m B_2[P_{\{n,m\}}] = \frac{3}{20}n+\frac{1}{6}mn$$

$$[9.]H_b[P_{\{n,m\}}] = \frac{11}{126}n+\frac{4}{5}mn.$$

Proof. Let $M(P\{n,m\},x,y)$ $=(2n)x^3y^3+(2n)x^3y^4+(2mn-5n)x^4y^4$

1. The atom-bond connectivity index

$$S_y^{\frac{1}{2}}g(x,y) = \frac{2}{\sqrt{3}}nx^3y^3+nx^3y^4+\frac{(2mn-5n)}{2}x^4y^4$$

$$S_x^{\frac{1}{2}}S_y^{\frac{1}{2}}g(x,y) = \frac{2}{3}nx^3y^3+\frac{1}{\sqrt{3}}nx^3y^4+\frac{(2mn-5n)}{4}x^4y^4$$

$$JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}g(x,y) = \frac{2}{3}nx^6+\frac{1}{\sqrt{3}}nx^7+\frac{(2mn-5n)}{4}x^8$$

$$Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}g(x,y) = \frac{2}{3}nx^4+\frac{1}{\sqrt{3}}nx^5+\frac{(2mn-5n)}{4}x^6$$

$$D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}g(x,y) = \frac{4}{3}nx^4+\sqrt{\frac{5}{3}}nx^5+\frac{\sqrt{6}}{4}(2mn-5n)x^6$$

$$ABC[H_{(m,n)}] = D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}[g(x,y)]_{x=1}$$

$$= (16+4\sqrt{15}-15\sqrt{6})\frac{n}{12}+\frac{\sqrt{6}}{2}mn$$

2. The geometric arithmetic index

$$D_y^{\frac{1}{2}}[g(x,y)] = 2\sqrt{3}nx^3y^3+4nx^3y^4+(4mn-10n)x^4y^4$$

$$D_x^{\frac{1}{2}}D_y^{\frac{1}{2}}[g(x,y)] = 6nx^3y^3+4\sqrt{3}nx^3y^4+(8mn-20n)x^4y^4$$

$$JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}[g(x,y)] = 6nx^6+4\sqrt{3}nx^7+(8mn-20n)x^8$$

$$S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}[g(x,y)] = nx^6+\frac{4\sqrt{3}}{7}nx^7+\frac{(2mn-5n)}{2}x^8$$

$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}[g(x,y)] = 2nx^6+\frac{8\sqrt{3}}{7}nx^7+(2mn-5n)x^8$$

$$GA[H_{(m,n)}] = 2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}[g(x,y)]_{x=1} = (\frac{-21+8\sqrt{3}}{7})n+2mn$$

3. The first K-Banhatti index

$$D_xg(x,y) = 6nx^3y^3+6nx^3y^4+(8mn-20n)x^4y^4$$

$$D_yg(x,y) = 6nx^3y^3+8nx^3y^4+(8mn-20n)x^4y^4$$

$$(D_x+D_y)[g(x,y)] = 12nx^3y^3+14nx^3y^4+(16mn-40n)x^4y^4$$

$$(D_x+D_y)g(x,y)_{x=y=1} = -14n+16mn$$

$$Jg(x,y) = 2nx^6+2nx^7+(2mn-5n)x^8$$

$$Q_{-2}Jg(x,y) = 2nx^4+2nx^5+(2mn-5n)x^6$$

$$D_xQ_{-2}Jg(x,y) = 8nx^4+10nx^5+(12mn-30n)x^6$$

$$2D_xQ_{-2}Jg(x,y) = 16nx^4+20nx^5+(24mn-60n)x^6$$

$$B_1[H_{(m,n)}] = (D_x+D_y+2D_xQ_{-2}J)[g(x,y)]_{x=y=1}$$

$$= -38n+40mn$$

4. The second K Banhatti index

$(D_x+D_y)[g(x,y)]=12nx^3y^3+14nx^3y^4+(16mn-40n)x^4y^4$

$J(D_x+D_y)g(x,y)=12nx^6+14nx^7+(16mn-40n)x^8$

$Q_{-2}J(D_x+D_y)g(x,y)=12nx^4+14nx^5+(16mn-40n)x^6$

$D_xQ_{-2}J(D_x+D_y)g(x,y)=48nx^4+70nx^5+(96mn-240n)x^6$

$B_2[H_{(m,n)}]=D_xQ_{-2}J(D_x+D_y)[g(x,y)]_{x=1}=-122n+96mn$

5. The first K-hyper Banhatti index

$D_x^2g(x,y)=18nx^3y^3+18nx^3y^4+(32mn-80n)x^4y^4$

$D_y^2g(x,y)=18nx^3y^3+32nx^3y^4+(32mn-80n)x^4y^4$

$(D_x^2+D_y^2)g(x,y)=36nx^3y^3+50nx^3y^4+(64mn-160n)x^4y^4$

$2D_xQ_{-2}J(D_x+D_y)g(x,y)=96nx^4+140nx^5+(192mn-480n)x^6$

$Q_{-2}Jg(x,y)=2nx^4+2nx^5+(2mn-5n)x^6$

$D_x^2Q_{-2}Jg(x,y)=32nx^4+50nx^5+(72mn-180n)x^6$

$2D_x^2Q_{-2}Jg(x,y)=64nx^4+100nx^5+(144mn-360n)x^6$

$$HB_1[H_{(m,n)}]=(D_x^2+D_y^2+2D_x^2Q_{-2}J+2D_xQ_{-2}J(D_x+D_y))[g(x,y)]_{x=y=1}$$

$HB_1[H_{(m,n)}]=-514n+400mn$

6. The second K-hyper Banhatti index

$(D_x^2+D_y^2)g(x,y)=36nx^3y^3+50nx^3y^4+(64mn-160n)x^4y^4$

$J(D_x^2+D_y^2)g(x,y)=36nx^6+50nx^7+(64mn-160n)x^8$

$Q_{-2}J(D_x^2+D_y^2)g(x,y)=36nx^4+50nx^5+(64mn-160n)x^6$

$$D_x^2Q_{-2}J(D_x^2+D_y^2)g(x,y)=576nx^4+1250nx^5+(2304mn-5760n)x^6$$

$HB_2[H_{(m,n)}]=D_x^2Q_{-2}J(D_x^2+D_y^2)[g(x,y)]_{x=1}$

$HB_2[H_{(m,n)}]=-3934n+2304mn$

7. Modified first K-Banhatti index

$L_xg(x,y)=2nx^6y^3+2nx^6y^4+(2mn-5n)x^8y^4$

$L_yg(x,y)=2nx^3y^6+2nx^3y^8+(2mn-5n)x^4y^8$

$J(L_x+L_y)g(x,y)=4nx^9+2n(x^{10}+x^{11})+(4mn-10n)x^{12}$

$Q_{-2}J(L_x+L_y)g(x,y)=4nx^7+2n(x^8+x^9)+(4mn-10n)x^{10}$

$S_xQ_{-2}J(L_x+L_y)g(x,y)=\frac{4}{7}nx^7+\frac{1}{4}nx^8+\frac{2}{9}nx^9+\frac{(2mn-5n)}{5}x^{10}$

$mB_1[H_{(m,n)}]=S_xQ_{-2}J(L_x+L_y)[g(x,y)]_{x=1}=\frac{11}{252}n+\frac{2}{5}mn$

8. Modified second K-Banhatti index

$S_yg(x,y)=\frac{2}{3}nx^3y^3+\frac{1}{2}nx^3y^4+\frac{(2mn-5n)}{4}x^4y^4$

$S_xg(x,y)=\frac{2}{3}nx^3y^3+\frac{2}{3}nx^3y^4+\frac{(2mn-5n)}{4}x^4y^4$

$J(S_x+S_y)g(x,y)=\frac{4}{3}nx^6+\frac{7}{6}nx^7+\frac{(2mn-5n)}{2}x^8$

$Q_{-2}J(S_x+S_y)g(x,y)=\frac{4}{3}nx^4+\frac{7}{6}nx^5+\frac{(2mn-5n)}{2}x^6$

$S_xQ_{-2}J(S_x+S_y)g(x,y)=\frac{1}{3}nx^4+\frac{7}{30}nx^5+\frac{(2mn-5n)}{12}x^6$

$mB_2[H_{(m,n)}]=S_xQ_{-2}J(S_x+S_y)[g(x,y)]_{x=1}=\frac{3}{20}n+\frac{1}{6}mn$

9. Harmonic K-Banhatti index

$S_xQ_{-2}J(L_x+L_y)g(x,y)=\frac{4}{7}nx^7+\frac{1}{4}nx^8+\frac{2}{9}nx^9+\frac{(2mn-5n)}{5}x^{10}$

$2S_xQ_{-2}J(L_x+L_y)g(x,y)=\frac{8}{7}nx^7+\frac{1}{2}nx^8+\frac{4}{9}nx^9+\frac{(4mn-10n)}{5}x^{10}$

$H_b[H_{(m,n)}]=2S_xQ_{-2}J(L_x+L_y)[g(x,y)]_{x=1}=\frac{11}{126}n+\frac{4}{5}mn$

Graphical analysis

Figure 2 is a graphical representation of topological indices of generalized prism network *P{n,m}*.

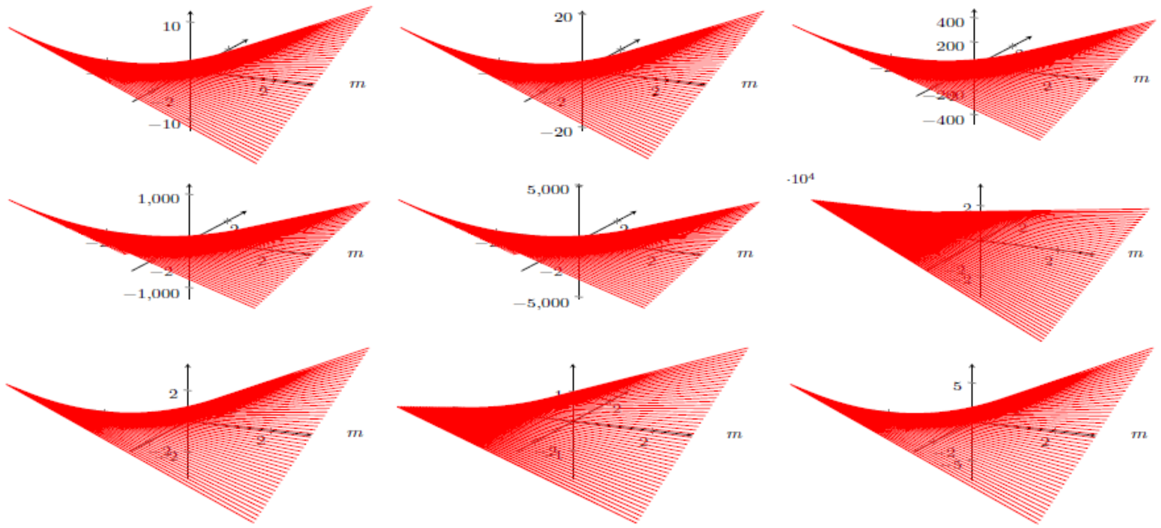

FIGURE 2 The plot of topological indices of generalized prism network *P{n,m}*.

Conclusion

In this article, we computed some degree-based topological indices for generalized prism P{n,m} from M-polynomial, accompanied by graphical representation.

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