Received: 17 December 2020 | Revised: 02 February 2021 | Accepted: 17 February 2021
DOI: 10.22034/ecc.2021.269116.1125
Eurasian Chemical Communications SAMI
FULL PAPER

Degree based topological indices of tadpole graph *via* M-polynomial

Faryal Chaudhry[a] |Muhammad Ehsan[a] |Farkhanda Afzal[b] |Mohammad Reza Farahani[c,*] | Murat Cancan[d] |Idris Çiftçi[d]

[a]*Department of Mathematics and Statistics, The University of Lahore, Lahore, 54000, Pakistan*
[b]*MCS, National University of Sciences and Technology, Islamabad, Pakistan*
[c]*Department of Applied Mathematics, Iran University of Science and Technology, Tehran, Iran*
[d]*Faculty of Education, Van Yuzuncu Yıl University, Zeve Campus, Tuşba, 65080, Van, Turkey*

***Corresponding Author:**
Farkhanda Afzal
Email: farkhanda@mcs.edu.pk
Tel.: 00923310046177

Chemical graph theory has an important impact on the development of the chemical sciences. A chemical graph is a graph that is produced from some molecular structure by applying some graphical operations. The demonstration of chemical compounds and chemical networks with the M-polynomials is a revolution and the M-polynomial of different molecular structures contributes to evaluating many topological indices. In this paper we worked out M-Polynomial and topological indices of the tadpole graph, then we recovered numerous topological indices using the M-polynomials.

KEYWORDS

Molecular graph; nanotube; topological indices; M-polynomial; Tadpole.

Introduction

The word "graph" was firstly used by *James J. Sylvester* (1814-1897) in 1878. In mathematics, graph theory is nowadays one of the rapidly growing branches. Graph theory has also found common application in various fields such as physics, biology, operations research, optimization theory, statistical mechanics, computer science, and engineering and even in chemistry. One of the most important sub fields of mathematical chemistry is chemical graph theory which was originated by *Ante Graovac, Milan Randić, Alexander Balaban, Haruo Hosoya, Nenad Trinajstić* and *Ivan Gutman* [27-34].

In chemical graph theory, the description of the molecule is generally characterized by graphs, which are identified by their vertices and edges. These graphs are set up from chemical molecules in which atoms convert vertices and bonds turn into edges of the graph. The topological index used for a molecular graph is the ultimate conclusion of mathematical and logical operations which transform chemical facts of the molecule into a suitable number. The Topological descriptors are derived from hydrogen-suppressed molecular graphs.

In theoretical chemistry several topological indices are presented to check the properties of molecules which are distributed in three classes distance dependent, degree dependent and counting related topological indices. A degree depended topological indices are calculated with the support of the degrees of vertices of chemical graph.

The topological indices calculated in this paper are atom bond connectivity index, symmetric division index, first and second K-Banhatti index, first and second K-hyper Banhatti index, modified first and second K-Banhatti index, and harmonic K-Banhatti index.

In the paper, all graphs are assumed selected simple and connected. We calculated M-polynomials of graphs and relying on this, we determined topological indices. These graphs are useful to understand the moving behavior of topological indices concerning the structure of a molecule. In this article, we computed a closed-form of some degree-based topological indices of tadpole by using an M-polynomial.

M-polynomial

An algebraic polynomial can also explain the behavior of the molecular structure. M-polynomial is also graph representative mathematical object. With the help of M-polynomial, we calculated many degree dependent topological invariant present in Table 3.

For a graph G, the M-polynomial presented in 2015[15] is defined as:

$$M(G,x,y)=\sum_{\delta\le i\le j\le\Delta} m_{ij}(G)x^i y^j$$

Where $\delta=min\{d_v/v\in V(G)\}$, $\Delta=max\{d_v/v\in V(G)\}$ and $m_{ij}(G)$ is the number of edges vu∈E(G) such that $\{d_v;d_u\}=\{i;j\}$. M-polynomial of many graphs are introduced [1,2,14,16,17,29,30] in the past. In this work we computed M-polynomials and topological indices of *T(n;m)*.

Tadpole graph

The Tadpole graph *T(n;m)*, presented in Figures 1, 2, 3 can be measured as a structure obtained by fusing six segments of linear polyacenes into a closed loop.

TABLE 1 Edge partition of Tadpole graph *T(n;m)*

$(d_u;d_v)$	Number of Edges
(1,2)	*1*
(2,2)	*(n+m -4)*
(2,3)	*3*
Total edges	*(n+m-4)+4*

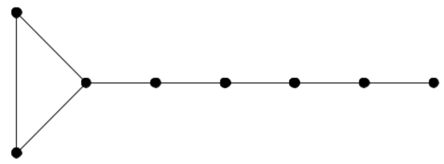


FIGURE 1 Tadpole graph T (3;5)

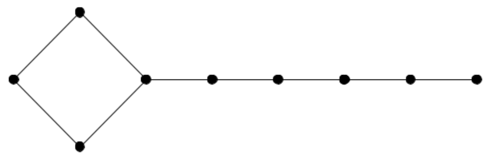


FIGURE 2 Tadpole graph T (4;5)

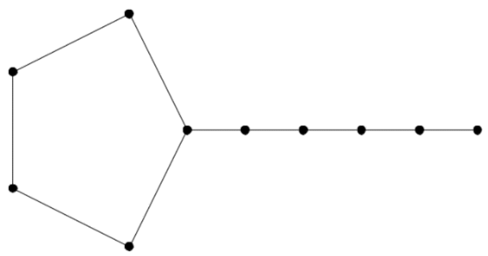


FIGURE 3 Tadpole graph T (5;5)

Topological indices

A topological index is a numeric real number related to the topology of the molecular graph. There are many classes of molecular topological invariants but degree dependent topological indices have an important role in the field of chemical graph theory. These degree dependent indices are computed on the basis of degrees of vertices of molecular graph. Usually carbon-hydrogen bond is surpassed during the study of topological indices because this bond does not have any effect on the topological properties of the molecular compound. These topological indices have many applications in QSAR and QSPR studies. Table 2 shows some important degree-based topological indices.

The following table tells some well-known degree -based topological indices compute via M-Polynomial.

TABLE 2 Degree-based topological indices

Atom-bond connectivity index [28]	$ABC[T(n,m)] = \sum_{uv\in E(T(n,m))} \sqrt{\frac{d_u + d_v - 2}{d_u.d_v}}$
Geometric arithmetic index [34]	$GA[T(n,m)] = \sum_{uv\in E(T(n,m))} \frac{2\sqrt{d_u.d_v}}{d_u + d_v}$
First K Banhatti index [31]	$B_1[T(n,m)] = \sum_{uv\in E(T(n,m))} (d_u + d_{uv})$
Second K Banhatti index [31]	$B_2[T(n,m)] = \sum_{uv\in E(T(n,m))} (d_u.d_{uv})$
First K hyper Banhatti index [32]	$HB_1[T(n,m)] = \sum_{uv\in E(T(n,m))} (d_u + d_{uv})^2$
Second K hyper Banhatti index [32]	$HB_2[T(n,m)] = \sum_{uv\in E(T(n,m))} (d_u.d_{uv})^2$
Modified first K Banhatti index [33]	${}^mB_1[T(n,m)] = \sum_{uv\in E(T(n,m))} \frac{1}{d_u + d_{uv}}$
Modified second K Banhatti index [33]	${}^mB_2[T(n,m)] = \sum_{uv\in E(T(n,m))} \frac{1}{d_u.d_{uv}}$
Harmonic K Banhatti index [33]	$H_b[T(n,m)] = \sum_{uv\in E(T(n,m))} \frac{2}{d_u + d_{uv}}$

TABLE 3 Degree dependent topological indices [2]

Topological index	Derivation from M(G;x,y)
Atom-bond connectivity index	$ABC[T(n,m)] = D_x^{\frac{1}{2}} Q_{-2} J S_x^{\frac{1}{2}} S_y \frac{1}{2}[f(x,y)]_{x=1}$
Geometric arithmetic index	$GA[T(n,m)] = 2S_x J D_x^{\frac{1}{2}} D_y^{\frac{1}{2}} [f(x,y)]_{x=1}$
First K Banhat[[ti index	$B_1[T(n,m)] = (D_x + D_y + 2D_x Q_{-2} J)[f(x,y)]_{x=y=1}$
Second K Banhatti index	$B_2[T(n,m)] = D_x Q_{-2} J(D_x + D_y)[f(x,y)]_{x=1}$
First K hyper Banhatti index	$HB_1[T(n,m)] = (D_x^2 + D_y^2 + 2D_x^2 Q_{-2} J + 2D_x Q_{-2} J(D_x + D_y))[f(x,y)]_{x=y=1}$
Second K hyper Banhatti index	$HB_2[T(n,m)] = D_x^2 Q_{-2} J(D_x^2 + D_y^2)[f(x,y)]_{x=1}$
Modified first K Banhatti index	${}^mB_1[T(n,m)] = S_x Q_{-2} J(L_x + L_y)[f(x,y)]_{x=1}$
Modified second K Banhatti index	${}^mB_2[T(n,m)] = S_x Q_{-2} J(S_x + S_y)[f(x,y)]_{x=1}$
Harmonic K Banhatti index	$H_b[T(n,m)] = 2S_x Q_{-2} J(L_x + L_y)[f(x,y)]_{x=1}$

Where the operator used is defined as

$$D_x f(x,y)=x\frac{\partial(f(x,y))}{\partial x}, D_y f(x,y)=y\frac{\partial(f(x,y))}{\partial y},$$

$$L_x f(x,y)=f(x^2,y), L_y f(x,y)=f(x,y^2),$$

$$S_x f(x,y)=\int_0^x \frac{f(t,y)}{t}dt, S_y f(x,y)=\int_0^y \frac{f(x,t)}{t}dt,$$

$$Jf(x,y)=f(x,x), Q_\alpha f(x,y)=x^\alpha f(x,y),$$

$$D_x^{\frac{1}{2}} f(x,y)=\sqrt{x\frac{\partial(f(x,y))}{\partial x}}\cdot\sqrt{f(x,y)},$$

$$D_y^{\frac{1}{2}} f(x,y)=\sqrt{y\frac{\partial(f(x,y))}{\partial y}}\cdot\sqrt{f(x,y)},$$

$$S_x^{\frac{1}{2}} f(x,y)=\sqrt{\int_0^x \frac{f(t,y)}{t}dt}\cdot\sqrt{f(x,y)},$$

$$S_y^{\frac{1}{2}} f(x,y)=\sqrt{\int_0^y \frac{f(x,t)}{t}dt}\cdot\sqrt{f(x,y)}.$$

M-Polynomial of Tadpole graph

Theorem 6.1. If tadpole graph is symbolized by *T(n ; m)* then for n,m≥3, M-polynomial of *T(n ; m)* is *M[T(n; m);x;y]=xy²+(n+m-4)x²y²+3x²y³.*

Proof: Let *T(n;m)* be a Tadpole then by using Figure and Table the Edge partition of Tadpole graph *T(n;m)* is

$$E_{1,2}(T(n,m))=\{e=uv\in T(n,m):d_u=1,d_v=2\}$$

$$\Rightarrow|E_{1,2}T(n,m)|=1,$$

$$E_{2,2}(T(n,m))=\{e=uv\in T(n,m):d_u=2,d_v=2\}$$

$$\Rightarrow|E_{2,2}T(n,m)|=(n+m-4),$$

$$E_{2,3}(T(n,m))=\{e=uv\in T(n,m):d_u=2,d_v=3\}$$

$$\Rightarrow|E_{2,3}T(n,m)|=3.$$

The following result is obtained by applying the interpretation of M-polynomial

$$M(T(n,m);x,y)=\sum_{\delta\le i\le j\le\Delta} m_{ij}(T(n,m))x^i y^j,$$

$$M(T(n,m);x,y)=\sum_{1\le i\le j\le 3} m_{ij}(T(n,m))x^i y^j,$$

$$M(T(n,m);x,y)=\sum_{1\le 2} m_{12}(T(n,m))xy^2$$

$$+\sum_{2\le 2} m_{22}(T(n,m))x^2y^2+\sum_{2\le 3} m_{23}(T(n,m)x^2y^3,$$

$$M(T(n,m);x,y)=|E_{1,2}|xy^2+|E_{2,2}|x^2y^2+|E_{2,3}|x^2y^3,$$

$$M(T(n,m);x,y)=xy^2+(n+m-4)x^2y^2+3x^2y^3$$

The structure of M-polynomial of T(n,m) is shown in Figure 4.

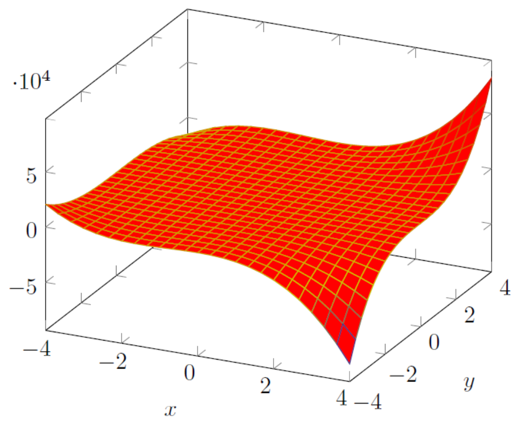


FIGURE 4 3D sketch of the M-polynomial of the Tadpole graph *T(n;m)* for n=m=4

Topological Indices of Tadpole Graph

Theorem 7.1. Let *T(n;m)* be a Tadpole M[*T(n;m)*;x;y]=xy²+(n+m-4)x²y²+3x²y³
Then:

$$[1.]ABC[T(n,m)]=\frac{1}{\sqrt{2}}(n+m)$$

$$[2.]GA[T(n,m)]=(n+m)+\left(\frac{18\sqrt{6}+10\sqrt{2}}{15}-4\right)$$

$$[3.]B_1[T(n,m)]=8(n+m)+6$$

$$[4.]B_2[T(n,m)]=8(n+m)+16$$

$$[5.]HB_1[T(n,m)]=32(n+m)+68$$

$$[6.]HB_2[T(n,m)]=32(n+m)+228$$

$$[7.]^m B_1[T(n,m)]=\frac{1}{2}(n+m)-\frac{1}{15}$$

$$[8.]^m B_2[T(n,m)]=\frac{1}{2}(n+m)+\frac{1}{3}$$

$$[9.]H_b[T(n,m)]=(n+m)-\frac{2}{15}$$

Proof: Let *M[T(n;m);x;y]=f(x;y)=xy²+(n+m-4)x²y²+3x²y³*

1. The atom-bond connectivity index

$$f(x,y)=xy^2+(n+m-4)x^2y^2+3x^2y^3$$

$$S_y^{\frac{1}{2}} f(x,y)=\frac{1}{\sqrt{2}}xy^2+\frac{1}{\sqrt{2}}(n+m-4)x^2y^2+\frac{3}{\sqrt{3}}x^2y^3$$

$$S_x^{\frac{1}{2}}S_y^{\frac{1}{2}} f(x,y)=\frac{1}{\sqrt{2}}xy^2+\frac{1}{2}(n+m-4)x^2y^2+\frac{3}{\sqrt{6}}x^2y^3$$

$$JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}f(x,y)=\frac{1}{\sqrt{2}}x^3+\frac{1}{2}(n+m-4)x^4+\frac{3}{\sqrt{6}}x^5$$

$$Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}f(x,y)=\frac{1}{\sqrt{2}}x+\frac{1}{2}(n+m-4)x^2+\frac{3}{\sqrt{6}}x^3$$

$$D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}f(x,y)=\frac{1}{\sqrt{2}}x+\frac{1}{\sqrt{2}}(n+m-4)x^2+\frac{3}{\sqrt{2}}x^3$$

$$ABC[T(n,m)]=D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}[f(x,y)]_{x=1}=\frac{1}{\sqrt{2}}(n+m)$$

2. The geometric arithmetic index

$$f(x,y)=xy^2+(n+m-4)x^2y^2+3x^2y^3$$

$$D_y^{\frac{1}{2}}f(x,y)=\sqrt{2}xy^2+\sqrt{2}(n+m-4)x^2y^2+3\sqrt{3}x^2y^3$$

$$D_x^{\frac{1}{2}}D_y^{\frac{1}{2}}f(x,y)=\sqrt{2}xy^2+2(n+m-4)x^2y^2+3\sqrt{6}x^2y^3$$

$$JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}f(x,y)=\sqrt{2}x^3+2(n+m-4)x^4+3\sqrt{6}x^5$$

$$S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}f(x,y)=\frac{\sqrt{2}}{3}x^3+\frac{1}{2}(n+m-4)x^4+\frac{3\sqrt{6}}{5}x^5$$

$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}f(x,y)=\frac{2\sqrt{2}}{3}x^3+(n+m-4)x^4+\frac{6\sqrt{6}}{5}x^5$$

$$GA[T(n,m)]=2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}[f(x,y)]_{x=1}=(n+m)+(\frac{18\sqrt{6}+10\sqrt{2}}{15}-4)$$

3. The first K Banhatti index

$$f(x,y)=xy^2+(n+m-4)x^2y^2+3x^2y^3$$

$$D_xf(x,y)=xy^2+2(n+m-4)x^2y^2+6x^2y^3$$

$$D_yf(x,y)=2xy^2+2(n+m-4)x^2y^2+9x^2y^3$$

$$(D_x+D_y)f(x,y)=3xy^2+4(n+m-4)x^2y^2+15x^2y^3$$

$$(D_x+D_y)f(x,y)_{x=y=1}=4(n+m-4)+18Jf(x,y)$$

$$=x^3+(n+m-4)x^4+3x^5$$

$$Q_{-2}Jf(x,y)=x+(n+m-4)x^2+3x^3$$

$$D_xQ_{-2}Jf(x,y)=x+2(n+m-4)x^2+9x^3$$

$$2D_xQ_{-2}Jf(x,y)=2x+4(n+m-4)x^2+18x^3$$

$$2D_xQ_{-2}Jf(x,y)_{x=1}=4(n+m-4)+20$$

$$B_1[T(n,m)]=(D_x+D_y+2D_xQ_{-2}J)[f(x,y)]_{x=y=1}=8(n+m)+6$$

4. The second K Banhatti index

$$f(x,y)=xy^2+(n+m-4)x^2y^2+3x^2y^3$$

$$D_xf(x,y)=xy^2+2(n+m-4)x^2y^2+6x^2y^3$$

$$D_yf(x,y)=2xy^2+2(n+m-4)x^2y^2+9x^2y^3$$

$$(D_x+D_y)f(x,y)=3xy^2+4(n+m-4)x^2y^2+15x^2y^3$$

$$J(D_x+D_y)f(x,y)=3x^3+4(n+m-4)x^4+15x^5$$

$$Q_{-2}J(D_x+D_y)f(x,y)=3x+4(n+m-4)x^2+15x^3$$

$$D_xQ_{-2}J(D_x+D_y)f(x,y)=3x+8(n+m-4)x^2+45x^3$$

$$B_2[T(n,m)]=D_xQ_{-2}J(D_x+D_y)[f(x,y)]_{x=1}=8(n+m)+16$$

5. The first K hyper Banhatti index

$$f(x,y)=xy^2+(n+m-4)x^2y^2+3x^2y^3$$

$$D_x^2f(x,y)=xy^2+4(n+m-4)x^2y^2+12x^2y^3$$

$$D_y^2f(x,y)=4xy^2+4(n+m-4)x^2y^2+27x^2y^3$$

$$(D_x^2+D_y^2)f(x,y)=5xy^2+8(n+m-4)x^2y^2+39x^2y^3$$

$$(D_x^2+D_y^2)f(x,y)_{x=y=1}=8(n+m-4)+44$$

$$Jf(x,y)=x^3+(n+m-4)x^4+3x^5$$

$$Q_{-2}Jf(x,y)=x+(n+m-4)x^2+3x^3$$

$$D_x^2Q_{-2}Jf(x,y)=x+4(n+m-4)x^2+27x^3$$

$$2D_x^2Q_{-2}Jf(x,y)=2x+8(n+m-4)x^2+54x^3$$

$$2D_x^2Q_{-2}J[f(x,y)]_{x=1}=8(n+m-4)+56$$

$$D_xf(x,y)=xy^2+2(n+m-4)x^2y^2+6x^2y^3$$

$$D_yf(x,y)=2xy^2+2(n+m-4)x^2y^2+9x^2y^3$$

$$(D_x+D_y)f(x,y)=3xy^2+4(n+m-4)x^2y^2+15x^2y^3$$

$$J(D_x+D_y)f(x,y)=3x^3+4(n+m-4)x^4+15x^5$$

$$Q_{-2}J(D_x+D_y)f(x,y)=3x+4(n+m-4)x^2+15x^3$$

$$D_xQ_{-2}J(D_x+D_y)f(x,y)=3x+8(n+m-4)x^2+45x^3$$

$$2D_xQ_{-2}J(D_x+D_y)f(x,y)=6x+16(n+m-4)x^2+90x^3$$

$$2D_xQ_{-2}J(D_x+D_y)[f(x,y)]_{x=1}=16(n+m-4)+96$$

$$HB_1[T(n,m)]=(D_x^2+D_y^2+2D_x^2Q_{-2}J$$
$$+2D_xQ_{-2}J(D_x+D_y))[f(x,y)]_{x=y=1}=32(n+m)+68$$

6. The second K hyper Banhatti index

$$f(x,y)=xy^2+(n+m-4)x^2y^2+3x^2y^3$$

$$D_x^2f(x,y)=xy^2+4(n+m-4)x^2y^2+12x^2y^3$$

$$D_y^2f(x,y)=4xy^2+4(n+m-4)x^2y^2+27x^2y^3$$

$$(D_x^2+D_y^2)f(x,y)=5xy^2+8(n+m-4)x^2y^2+39x^2y^3$$

$$J(D_x^2+D_y^2)f(x,y)=5x^3+8(n+m-4)x^4+39x^5$$

$$Q_{-2}J(D_x^2+D_y^2)f(x,y)=5x+8(n+m-4)x^2+39x^3$$

$$D_x^2Q_{-2}J(D_x^2+D_y^2)f(x,y)=5x+32(n+m-4)x^2+351x^3$$

$$HB_2[T(n,m)]=D_x^2Q_{-2}J(D_x^2+D_y^2)[f(x,y)]_{x=1}=32(n+m)+228$$

7. Modified first K Banhatti index

$$f(x,y)=xy^2+(n+m-4)x^2y^2+3x^2y^3$$

$$L_xf(x,y)=x^2y^2+(n+m-4)x^4y^2+3x^4y^3$$

$$L_yf(x,y)=xy^4+(n+m-4)x^2y^4+3x^2y^6$$

$$(L_x+L_y)f(x,y)=x^2y^2+(n+m-4)x^4y^2+3x^4y^3$$
$$+xy^4+(n+m-4)x^2y^4+3x^2y^6$$

$$J(L_x+L_y)f(x,y)=x^4+x^5+2(n+m-4)x^6+3x^7+3x^8$$

$$Q_{-2}J(L_x+L_y)f(x,y)=x^2+x^3+2(n+m-4)x^4+3x^5+3x^6$$

$$S_xQ_{-2}J(L_x+L_y)f(x,y)=\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{2}(n+m-4)x^4+\frac{3}{5}x^5+\frac{1}{2}x^{6m}$$

$$B_1[T(n,m)]=S_xQ_{-2}J(L_x+L_y)[f(x,y)]_{x=1}=\frac{1}{2}(n+m)-\frac{1}{15}$$

8. Modified second K Banhatti index

$$f(x,y)=xy^2+(n+m-4)x^2y^2+3x^2y^3$$

$$S_x f(x,y)=xy^2+\frac{1}{2}(n+m-4)x^2y^2+\frac{3}{2}x^2y^3$$

$$S_y f(x,y)=\frac{1}{2}xy^2+\frac{1}{2}(n+m-4)x^2y^2+x^2y^3$$

$$(S_x+S_y)f(x,y)=\frac{3}{2}xy^2+(n+m-4)x^2y^2+\frac{5}{2}x^2y^3$$

$$J(S_x+S_y)f(x,y)=\frac{3}{2}x^3+(n+m-4)x^4+\frac{5}{2}x^5$$

$$Q_{-2}J(S_x+S_y)f(x,y)=\frac{3}{2}x+(n+m-4)x^2+\frac{5}{2}x^3$$

$$S_xQ_{-2}J(S_x+S_y)f(x,y)=\frac{3}{2}x+\frac{1}{2}(n+m-4)x^2+\frac{5}{6}x^{3m}$$

$$B_2[T(n,m)]=S_xQ_{-2}J(S_x+S_y)[f(x,y)]_{x=1}=\frac{1}{2}(n+m)+\frac{1}{3}$$

9. Harmonic K Banhatti index

$$f(x,y)=xy^2+(n+m-4)x^2y^2+3x^2y^3$$

$$L_x f(x,y)=x^2y^2+(n+m-4)x^4y^2+3x^4y^3$$

$$L_y f(x,y)=xy^4+(n+m-4)x^2y^4+3x^2y^6$$

$$(L_x+L_y)f(x,y)=(x^2y^2+(n+m-4)x^4y^2+3x^4y^3)+(xy^4+(n+m-4)x^2y^4+3x^2y^6)$$

$$J(L_x+L_y)f(x,y)=x^4+x^5+2(n+m-4)x^6+3x^7+3x^8$$

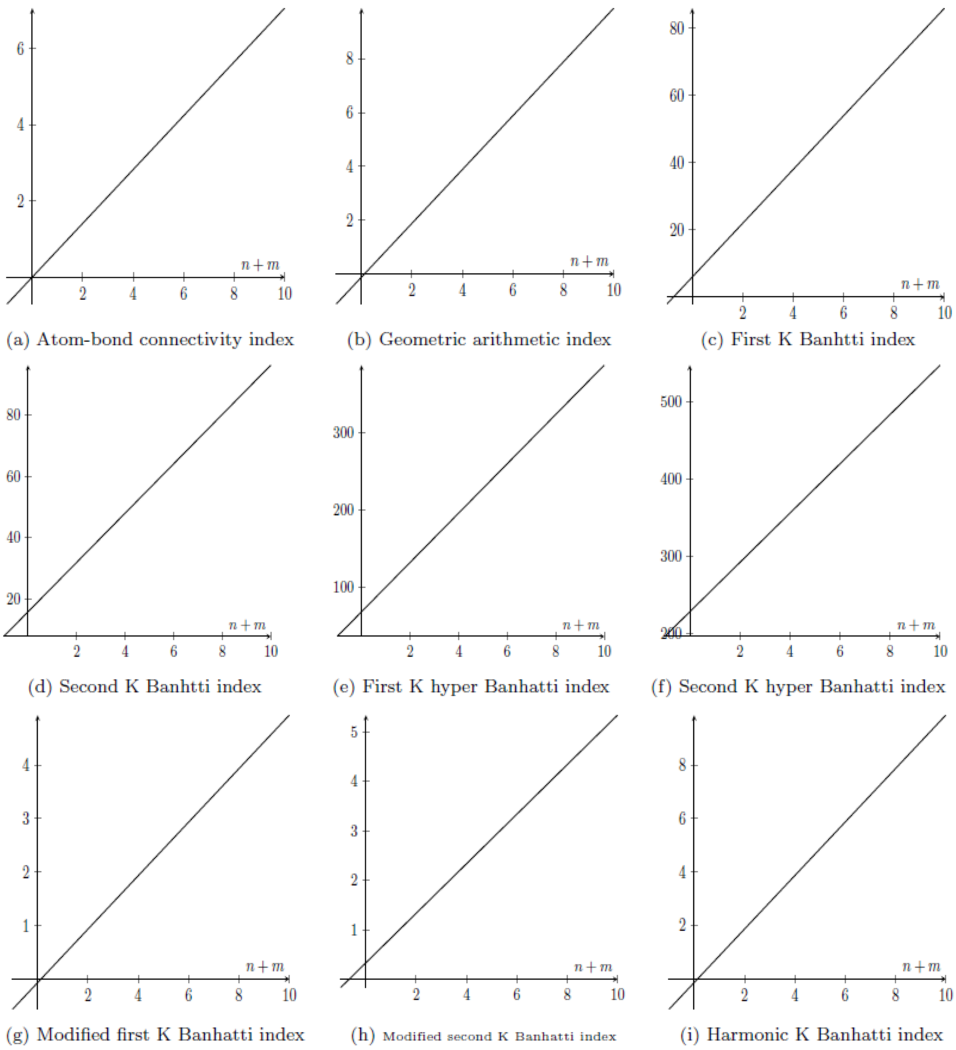
$$Q_{-2}J(L_x+L_y)f(x,y)=x^2+x^3+2(n+m-4)x^4+3x^5+3x^6$$

$$S_xQ_{-2}J(L_x+L_y)f(x,y)=\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{2}(n+m-4)x^4+\frac{3}{5}x^5+\frac{1}{2}x^6$$

$$2S_xQ_{-2}J(L_x+L_y)f(x,y)=x^2+\frac{2}{3}x^3+(n+m-4)x^4+\frac{6}{5}x^5+x^6$$

$$H_b[T(n,m)]=2S_xQ_{-2}J(L_x+L_y)[f(x,y)]_{x=1}=(n+m)-\frac{2}{15}\ \blacksquare$$

Figure 5 illustrates topological indices of *T(n;m)*. From graphs, we see the performance of the topological indices along parameter.



(a) Atom-bond connectivity index (b) Geometric arithmetic index (c) First K Banhtti index (d) Second K Banhtti index (e) First K hyper Banhatti index (f) Second K hyper Banhatti index (g) Modified first K Banhatti index (h) Modified second K Banhatti index (i) Harmonic K Banhatti index

FIGURE 5 The plot of topological indices 1 T(n;m)

Conclusion

We worked out closed form of M-polynomial for tadpole graph resulting various degree-based topological indices as well. Topological indices help to reduce the number of experiments. These topological indices can contribute to understand biological, chemical and physical characteristics of a molecule. So, topological index has a key role in representing the chemical structure of a molecule in a real number and is used to express the molecule which is tested. These results are helpful in understanding and forecasting the physiochemical properties for these chemical structures.

Acknowledgments

The authors extend their real appreciation to the reviewers for their insightful comments and technical suggestions to enhance quality of the article.

Orcid:

Farkhanda Afzal: *https://orcid.org/0000-0001-5396-7598*
Mohammad Reza Farahani: *https://orcid.org/0000-0003-2969-4280*
Murat Cancan: *https://orcid.org/0000-0002-8606-2274*
Idris Çiftçi: *https://orcid.org/0000-0002-2698-0807*

References

[1] D. Afzal, S. Hussain, M. Aldemir, M. Farahani, F. Afzal, *Eurasian Chem. Commun.*, **2020**, *2*, 1117-1125.

[2] F. Afzal, S. Hussain, D. Afzal, S. Razaq, *J. Inf. Opt. Sci.*, **2020**, *41*, 1061–1076.

[3] M. Cancan, S. Ediz, M.R. Farahani, *Eurasian Chem. Commun.*, **2020**, *2*, 641-645.

[4] A.Q. Baig, M. Naeem, W. Gao, J.B. Liu, *Eurasian Chem. Commun.*, **2020**, *2*, 634-640.

[5] F. Afzal, M.A. Razaq, D. Afzal, S. Hameed, *Eurasian Chem. Commun.*, **2020**, *2*, 652-662.

[6] M. Alaeiyan, C. Natarajan, G. Sathiamoorthy, M.R. Farahani, *Eurasian Chem. Commun.*, **2020**, *2*, 646-651.

[7] H. Yang, X. Zhang, *J. D. Math. Sci. and Cryp.*, **2018**, *21*, 1495-1507.

[8] M. Imran, S.A. Bokhary, S. Manzoor, M.K. Siddiqui, *Eurasian Chem. Communm.*, **2020**, *2*, 680-687.

[9] Z. Ahmad, M. Naseem, M.K. Jamil, Sh. Wang, M.F. Nadeem, *Eurasian Chem. Commun.*, **2020**, *2*, 712-721.

[10] Z. Ahmad, M. Naseem, M.K. Jamil, M.K. Siddiqui, M.F. Nadeem, *Eurasian Chem. Commun.*, **2020**, *2*, 663-671.

[11] M. Cancan, S. Ediz, M.R. Farahani, *Eurasian Chem. Commun.*, **2020**, *2*, 641-645.

[12] A.Q. Baig, M. Naeem, W. Gao, J.B. Liu, *Eurasian Chem. Commun.*, **2020**, *2*, 634-640.

[13] F. Afzal, M.A. Razaq, D. Afzal, S. Hameed, *Eurasian Chem. Commun.*, **2020**, *2*, 652-662.

[14] F. Afzal, S. Hussain, D. Afzal, S. Hameed, *Open Chem.,* **2020**, *18*, 1362–1369.

[15] F. Afzal, S. Hussain, D. Afzal, S. Razaq, *J. Inf. Opt. Sci.,* **2020**, *41*, 1061-1076.

[16] M. Cancan, D. Afzal, S. Hussain, A. Maqbool, F. Afzal, *J. Dis. Math. Sci. Cryp.,* **2020**, *23*, 1157-1171.

[17] M. Cancan, S. Ediz, H. Mutee-Ur-Rehman, D. Afzal, *J. Inf. Opt. Sci.*, **2020**, *41*, 1117-1131.

[18] M.R. Farahani, *Acta Chim. Slov.*, **2012**, *59*, 965-968,.

[19] Z. Shao, A.R Virk, M.S Javed, M.A Rehman, M.R. Farahani, *Open Appl. Sci. Let.*, **2019**, *2*, 01-11.

[20] M.R. Farahani, *World Appl. Sci. J.*, **2012**, *20*, 1248-1251.

[21] M.H.A. Siddiqui, M.R. Farahani, *Open J. Math. Anal.*, **2017**, *1*, 45-60.

[22] M.R. Farahani, *Int. J. Theor.l Chem.*, **2013**, *1*, 01-09.

[23] M.R. Farahani, *Chem. Phys. Res, J.*, **2013**, *6*, 27-33.

[24] M.R. Farahani, W. Gao. *J. Chem. Pharmaceutical Research*, **2015**, *7*, 535-539.

[25] M.R. Farahani, *J. Chem. Acta.*, **2013**, *2*, 22-25.

[26] W. Gao, M.R. Farahani, *J. Nanotech*., **2016**, *2016*, Article ID 3129561, 1-6.
[27] E. Deutsch, S. Klavžar, *Iran. J. Math. Chem*., **2015**, *6*, 93-102.
[28] E. Estrada, L. Torres, L. Rodrguez, I. Gutman, *Indian J. Chem*., **1998**, *37*, 849-855.
[29] S.M. Kang, W. Nazeer, W. Gao, D. Afzal, S.N. Gillani, *Open Chem.,* **2018**, *16*, 201-213.
[30] A.J.M. Khalaf, S. Hussain, D. Afzal, F. Afzal, A. Maqbool, *J. Dis. Math. Sci. Cryp.,* **2020,** *23*, 1217-1237.
[31] V. Kulli, *J. Com. Math. Sci*., **2016,** *7*, 213-218.
[32] V. Kulli, *Int. Res. J.Pure Algebra*, **2016**, *6*, 300-304.
[33] V. Kulli, *Int. J. Fuzzy Math. Arch.,* **2017**, *12*, 29-37.
[34] D. Vukičević, B. Furtula, *J. Math. Chem.*, **2009**, *46*, 1369-1376.

How to cite this article: Faryal Chaudhry, Muhammad Ehsan, Farkhanda Afzal*, Mohammad Reza Farahani, Murat Cancan, Idris Çiftçi. Degree based topological indices of tadpole graph *via* M-polynomial. *Eurasian Chemical Communications*, 2021, 3(3), 146-153. **Link:** http://www.echemcom.com/article_126645.html