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FULL PAPER

The first and second Zagreb polynomial and the forgotten polynomial of $C_m \times C_n$

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In this paper, the 1st and 2nd Zagreb polynomials and the forgotten polynomial of $C_m \times C_n$ were computed. Some degree-based topological indices such as 1st and 2nd multiple Zagreb indices, Hyper Zagreb index and the forgotten index or F-index of the given networks were also computed. In addition, we represented the outcome by graphical representation that describe the dependence of topological indices on the given parameters of polynomial structures.

KEYWORDS

Algebraic polynomial; topological descriptor; Zagreb indices; topological indices.

Introduction

A topological descriptor is a kind of a molecular topological invariant that is computed based on molecular structure of some chemical compound. A large number of chemical experiments look for a resolution of the chemical characteristics of compounds and drugs. Also, these chemical-based experiments validate that there is robust inherent correlation among the chemical characteristics of compounds, drugs and their molecular structures [1-7].

In the field of mathematical chemistry, topological index is a number associated with molecular graph of a compound that depends on topology of that compound and assists in deciding the properties of apprehensive compound. A topological index can be designed by restructuring any chemical structure into some quantity. Thus, these topological invariants are related with some physicochemical characteristics such as stability, strain energy and boiling point etc. These can be computed by means of definitions. Since its beginning in three to four decades ago, this issue has attracted many researchers from developing countries, because of its significance in the field of biochemistry, biotechnology, medical science and theoretical Chemistry [8-18].

Some basic definitions, related results and further information on topological indices can be obtained in the literature [19-31].

Main results

In this section, we provided main computational results. We computed the 1st and 2nd Zagreb polynomial, and the forgotten polynomial of $C_m \times C_n$ in this article. We also computed some degree-based topological indices such as 1st and 2nd multiple Zagreb indices, Hyper Zagreb index and the forgotten index or F-index for the given networks.

Theorem 1. Let $G \approx C_m \times C_n$ be the graph, the 1st Zagreb polynomial for this graph is

$$M_1(C_m \times C_n) = 2mnx^8.$$

Proof. Let $G \approx C_m \times C_n$ be the graph then the cardinality of vertex set is *mn* and edge set is 2mn. Then

$$M_1(G,x) = \sum_{uv \in E(G)} x^{(d_u + d_v)}$$

$$M_1(C_m \times C_n, x) = \sum_{uv \in E(C_m \times C_n)} x^{(d_u + d_v)}$$

since degree of all vertices is 4 so,

$$M_1(C_m \times C_n, x) = |E(C_m \times C_n)|\, x^{(4+4)} = 2mnx^8.$$

Theorem 2. Let $G \approx C_m \times C_n$ be the graph, the 2st Zagreb polynomial for this graph is

$$M_2(C_m \times C_n) = 2mnx^{16}.$$

Proof. Let $G \approx C_m \times C_n$ be the graph, the cardinality of vertex set is *mn* and edge set is 2mn. Then

$$M_2(G,x) = \sum_{uv \in E(G)} x^{(d_u \times d_v)}$$

$$M_2(C_m \times C_n, x) = \sum_{uv \in E(C_m \times C_n)} x^{(d_u \times d_v)}$$

since degree of all vertices is 4 so,

$$M_2(C_m \times C_n, x) = |E(C_m \times C_n)|\, x^{(4 \times 4)} = 2mnx^{16}.$$

Theorem 3. Let $G \approx C_m \times C_n$ be the graph, the forgotten polynomial for this graph is $F(C_m \times C_n) = 2mnx^{32}$.

Proof. Let $G \approx C_m \times C_n$ be the graph, the cardinality of vertex set is *mn* and edge set is 2mn. Then

$$F(G) = \sum_{uv \in E(G)} ((d_u)^2 + (d_v)^2)$$

$$F(C_m \times C_n) = \sum_{uv \in E(C_m \times C_n)} ((d_u)^2 + (d_v)^2)$$

$$= |E(C_m \times C_n)|((4)^2 + (4)^2)$$

$$= 2mnx^{32}$$

The graph of 1st Zagreb polynomial, 2nd Zagreb polynomial and forgotten polynomial for $C_8 \times C_7$ are shown in Figure 1. Below.

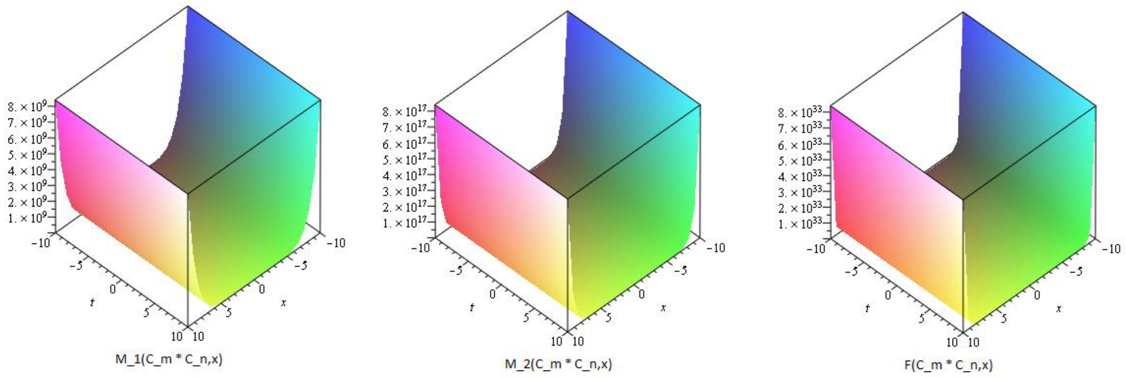

FIGURE 1 Graph of algebraic polynomials for $(C_8 \times C_7)$

Proposition: Let $G \approx C_m \times C_n$ be the graph then the hyper Zagreb index, 1st multiple Zagreb index, 2nd multiple Zagreb index and forgotten index are

$$HM(C_m \times C_n) = 132mn;$$

$$PM_1(C_m \times C_n) = 64^{mn};$$

$$PM_2(C_m \times C_n) = 256^{mn};$$

$$F(C_m \times C_n) = 32^{2mn}.$$

Proof. Let $G \approx C_m \times C_n$ be the graph then the cardinality of vertex set is *mn* and edge set is *2mn.* Then

$$HM(G) = \sum_{uv \in E(G)} (d_u + d_v)^2$$

$$HM(C_m \times C_n) = \sum_{uv \in E(C_m \times C_n)} (d_u + d_v)^2$$

$$= |E(C_m \times C_n)|(8)^2$$

$$= 132mn$$

by definition

$$PM1(G)=\prod_{uv\in E(G)}(d_u+d_v)$$

$$PM_1(C_m\times C_n)=\prod_{uv\in E(C_m\times C_n)}(d_u+d_v)$$

$$=(4+4)^{|E(C_m\times C_n)|}$$

$$=64mn.$$

by definition

$$PM_2(G)=\prod_{uv\in E(G)}(d_u\times d_v)$$

$$PM_2(C_m\times C_n)=\prod_{uv\in E(C_m\times C_n)}(d_u\times d_v)$$

$$PM_2(C_m\times C_n)=\prod_{uv\in E(C_m\times C_n)}(4\times 4)^{|E(C_m\times C_n)|}$$

$$=\prod_{uv\in E(C_m\times C_n)}256^{mn}.$$

$$F(C_m\times C_n)=\prod_{uv\in E(C_m\times C_n)}(d_u^2+d_v^2)$$

$$F(C_m\times C_n)=\prod_{uv\in E(C_m\times C_n)}(4^2+4^2)$$

$$=(16+16)^{|E(C_m\times C_n)|}=32^{2mn}.$$

3D plot for hyper Zagreb index, 1st Zagreb index, 2nd Zagreb index and forgotten index are shown in Figure 2.

TABLE 1 Edge partition of $C_m\times C_n$ based on degree of end vertices of each edge

(d_u; d_v)	**(2; 3)**	**(3; 3)**	**(3; 4)**	**(4; 4)**
Frequency	8	6+4t	10(t-1)t	$4t^2$+5t-3

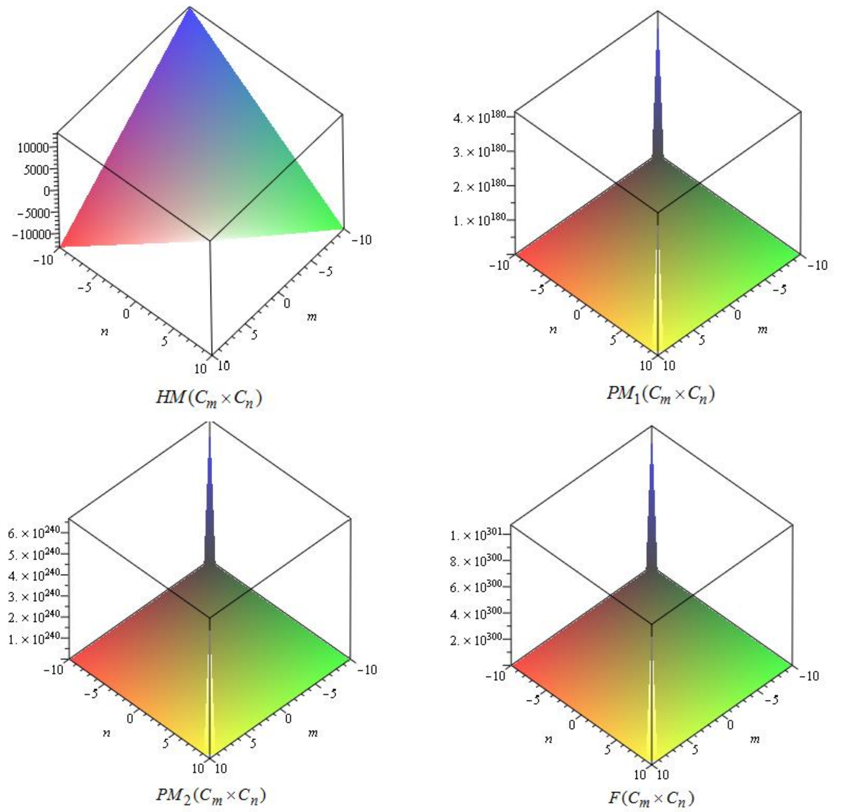

FIGURE 2 Graph of topological indices for ($C_m\times C_n$)

Conclusion

Graphs are indeed an important tool used for describing and representing algorithms, information related structures, networks, flows of electricity, social interaction and many other things in science and technology. Here, we computed several degree-based topological indices such as 1st multiple Zagreb index, 2nd multiple Zagreb index, Hyper Zagreb index and the forgotten index or F-index of these networks. These topological indices can help us in understanding of the chemical reactivity, biological problems and several physical features.

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