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FULL PAPER

Multiplicative degree-based topological indices and line graph of hex board graph

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Mathematical chemistry is the area of research in mathematics, in which problems of chemistry are solved by utilizing techniques of mathematics. In mathematical chemistry, a number is assigned to molecular graph of compound called topological index (TI) which depends on the topology of compound and helps us in deciding properties of concerned compound. TIs usually depend on the degree of vertices in a graph, distances and spectrum, among which degree depending TIs are studied extensively in recent years and have led to huge applications in theoretical chemistry, drugs formulation and pharmacy. This paper aimed to compute some degree depending TIs of Hex board networks and line graph of hex board networks. The generalized first and second multiplicative Zagreb indices (ZIs), multiplicative version of Atomic bond connectivity index (ABC) and generalized multiplicative Geometric Arithmetic index (GA) of Hex board and the line graph of Hex board networks were computed in this study.

KEYWORDS

Randić index; hexagonal network; line graph; topological index; Zagreb index; hex board.

Introduction

In Mathematical chemistry, we give graphical representation of compounds by taking atoms as vertices and bounds between atoms as edges. The number of vertices attached with a vertex x is called degree of x, which is very close to the valence in chemistry. Throughout this study, we mean M for the molecular graph, E for the edges and V for the vertices. It is important to mention here that all molecular graphs are connected and simple [1,2].

Usually, the hydrogen suppressed graphs are used in CGT because the hydrogen atoms and bounds, due to hydrogen atoms, do not affect the properties of molecules. The hydrogen stifled graphs are generally utilized in chemical graph theory on the grounds that the disregards of hydrogen molecule and their bonds cannot often be reason for any equivocalness. The molecular graph terribly simplifies the perplexing picture of particle by depicting just its constitution and neglecting the certain features (e.g. Geometry, Chirality, and Stereochemistry). Indeed, the basic picture of molecular graph empowers one to make valuable forecasts about physical and chemical properties of atoms. Since the expectations of properties and reactivates of particles are of prime enthusiasm to scientists, the advancement of chemical graph theory is consequently justified.

The TIs are numbers depending on the molecular graph and helpful in deciding the properties of the concerned molecular compound. We can consider TI as a function which assigns a real number to each molecular graph, which is used as descriptor of the concerned molecule. From the TIs, a variety of physical and chemical properties like heat of evaporation, heat of formation, boiling point, chromatographic retention, surface tension and vapor pressure of understudy molecular compound can be identified. A TI gives us 6 types of mathematical language to study a molecular graph. There are three types of TIs:

- Degree based TIs.
- Distance based TIs.
- Spectrum based TIs.

The first type of TI depends upon the degree of vertices, the second one depends upon the distance of vertices and the third type of TI depends upon the spectrum of graph.

Like TIs, polynomials also play an important role in chemistry. There are many polynomials in literature to define the molecular graph, for example, Hosoya polynomial, which is also known as Wiener polynomial, defined in [3]. It is assumed to be an important job in distance based TIs. Almost all distance based TIs can be recovered from this Hosoya polynomial. Motivated by the Hosoya polynomial, Deutsch et al. (2015) [4] introduced the M-polynomial, which plays a parallel role in computing with degree-based TIs. This polynomial is important due to the information it contains about degree-based TIs [5-10].

The graph network, especially honeycomb and hexagonal networks, plays an important role in biology and chemistry. Figure 1 is the graph of Hex board graph.

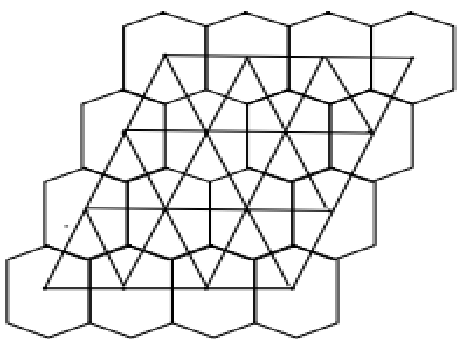

FIGURE 1 Hex board with center dots [11]

For a graph M, the line graph is denoted by L(M) is a graph obtained by taking edge of M as vertex and joining are vertices with an edge if the concerned edge in M has a common vertex [11]. The line graph of Hex board network is shown in Figure 2.

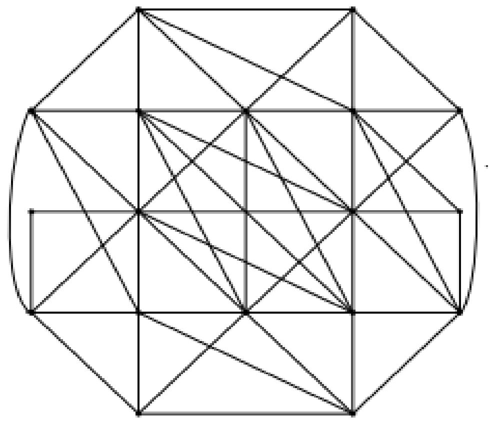

FIGURE 2 Line graph of hex board with center dots [11]

This paper aims to compute multiplicative version of degree depending TIs, for example ZIs, ABC and GA index for Hex board networks and line graph of Hex board networks. Firstly, we computed generalized version of these TIs and as immediate consequence, desired results were computed. This study was organized as follows: In the second section, we defined some multiplicative versions of TIs and highlighted their importance in chemistry. The third section contains methodology. The 4th section presents main computational results and finally in section 5, we demonstrated the conclusion.

Basic definitions and literature review

The very first TI was due to Wiener and is called Wiener index, which was initially named as path index and found huge applications in chemistry. Due to its applications, this index was the focus point of many researchers and hundreds of research studies have been written on this TI, for example [12, 13]. The first genuine degree depending TI is the Randić index (RI) which was given in 1975 by Milan Randić $R_{-1/2}(M)=\sum_{xy\in E(M)}\frac{1}{\sqrt{d_x d_y}}$.

For details about RI, please see [14-19].

Firstly, Randić named it as branching index, which was soon named as connectivity index and now a days it is called RI. The RI is the most popular degree based TIs and has been extensively studied by both mathematicians and chemists. Randić himself write two reviews and many papers and books on this topological invariant which are present in literature, few of which are [20-22]. Researchers recognized the importance of RI in drug design. Erdos and Bollbas, famous mathematicians of that time, investigated some hidden mathematical properties of RI. After that, RI has found its way into research and surge of publications have began. An unexpected mathematical quality of Randić index is discovered recently, telling us about the relation of this topological invariant with normalized Laplacian Matrix.

ZIs are defined for calculation of electronic energies, but soon it was observed that this index increased with increases in branching of the skeleton of carbon atoms. After 10 years, Balaban et al wrote a review article, declaring M_1 and M_2 among the degree based TIs and named them Zagreb Group Indices. The name Zagreb group indices soon changed into ZI and now M_1 and M_2 are abbreviated as first Zagreb index and second Zagreb index. In 1975, Gutman gave a remarkable identity and hence, these two indices are among the oldest degree depending descriptors and their properties are extensively investigated. The mathematical formulae of these indices are [23, 24]

$$M_1(M)=\sum_{x\in V(M)}(d_x)^2=\sum_{xy\in E(M)}(d_x+d_y),$$

$$M_2(M)=\sum_{xy\in E(M)}d_x\times d_y.$$

The multiplicative version of these ZIs are introduced in [25] as

$$II_1(M)=\prod_{x\in V(M)}(d_x)^2,$$

$$II_2(M)=\prod_{xy\in E(M)}d_x\cdot d_y,$$

Another TI is Narumi-Katayama [26], which is introduced as $NK(M)=\prod_{x\in V(M)}d_x$.

The TIs also attracts researchers due to application of them in chemistry and other sciences [27-30]. In [30], famous chemist, Gutman (2011) determined the trees for the maximum and minimum multiplicative ZIs. The idea of Gutman was extended in [30] and following index for trees were introduced: $W_1^s(M)=\prod_{x\in V(M)}(d_x)^s$.

One can observe that for specific values of parameter s, Narumi-Katayama and first multiplicative ZI can be obtained from above defined index respectively. Motivated by the success of multiplicative ZIs Eliasi et al. in [31] introduced another version of first multiplicative ZI as

$$II_1^*(M)=\prod_{xy\in E(M)}(d_x+d_y).$$

But the both version is different and new. Advancing the idea of indexing with the edge set, the multiple version of first and second hyper-ZIs [32] are characterized as

$$HII_1(M)=\prod_{xy\in E(M)}(d_x+d_y)^2,$$

$$HII_2(M)=\prod_{xy\in E(M)}(d_x\cdot d_y)^2.$$

Recently generalized versions of multiplicative ZIs are introduced in [33] by Kulli et al. as:

$$MZ_1^k(M)=\prod_{xy\in E(M)}(d_x+d_y)^k,$$

$$MZ_2^k(M)=\prod_{xy\in E(M)}(d_x\cdot d_y)^k.$$

Other well studied and well applied multiplicative indices are multiplicative versions of sum and product connectivity indices [34] with following mathematical formulae:

$$SCII(M)=\prod_{xy\in E(M)}\frac{1}{\sqrt{d_x+d_y}},$$

$$PCII(M)=\prod_{xy\in E(M)}\frac{1}{\sqrt{d_x\cdot d_y}}.$$

It can be seen that, for specific values of k in generalized multiplicative ZIs, we get other indices, for example:

- for k=1, we get multiplicative ZIs,
- for k=2, we get multiplicative hyper ZIs,
- for k=1/2, we get sum and product connectivity indices

Mathematical formulae for multiplicative *ABC, GA* and generalized GA can be defined as:

$$ABCII(M)=\prod_{xy\in E(M)}\sqrt{\frac{d_x+d_y-2}{d_x\cdot d_y}},$$

$$GAII(M)=\prod_{xy\in E(M)}\frac{2\sqrt{d_x\cdot d_y}}{d_x+d_y},$$

$$GA^kII(M)=\prod_{xy\in E(M)}\left(\frac{2\sqrt{d_x\cdot d_y}}{d_x+d_y}\right)^k.$$

For details about the applications of graph theory, chemical graph theory and TIs in chemistry, physics, biology and other areas, we refer to [35-42] and references therein.

Methodology

To compute our main results, firstly, we counted the number of edges and vertices in the Hex board graphs and line graph of hex board graphs. Secondly, we divided the edge set into different classes with respect to the degrees of vertices. From the edge partition, we computed our results by applying definitions.

Main results

Main computational results are given in this section. In section 4.1, generalized of first and second multiplicative ZIs, multiplicative of *ABC* and *GA* index of Hex board graph are computed. In section 4.2, generalized version of first and second multiplicative ZIs, multiplicative of *ABC* and *GA* index of line graph of Hex board graph are computed.

Multiplicative degree-based topological indices of Hex Board Graph

The following theorem is about the multiplicative generalized ZIs and *GA* index.

Theorem 1. Consider $M=H_n$ be Hex board graph. Then the multiplicative versions of generalized first ZI, multiplicative version of generalized second ZIx and generalized version of GA index of M are:

1. $$MZ_1^k(M)=(12)^{3km^2}\times\left(\frac{5^8}{2^{12}\times 3^{16}}\right)^{km}\left(\frac{3^{29}\times 7^4}{2^4\times 5^{20}}\right)^k.$$

2. $$MZ_2^\alpha(H_n)=(36)^{3\alpha k^2}\times\left(\frac{2^8}{3^{24}}\right)^{\alpha k}\left(\frac{3^{30}}{2^{36}}\right)^{\alpha}.$$

3. $$GA^\alpha II(H_n)=\left[\frac{2^7\times 3^{4k}\times 5^{10}}{2^{4k}\times 3^{10}\times 7^4\times 5^{8k}}\right]^{\alpha}.$$

Proof. Let $M=H_n$ be Hex board. The edge set of M has following six partitions [43]

$$E_1=\{xy\in E(M)|d_x=2,d_y=4\},$$
$$E_2=\{xy\in E(M)|d_x=3,d_y=4\},$$
$$E_3=\{xy\in E(M)|d_x=3,d_y=6\},$$
$$E_4=\{xy\in E(M)|d_x=4,d_y=4\},$$
$$E_5=\{xy\in E(M)|d_x=4,d_y=6\},$$
$$E_6=\{xy\in E(M)|d_x=6,d_y=6\},$$

Such that

$|E_1|=4,\ |E_2|=4,\ |E_3|=2,\ |E_4|=4m-10,$

$|E_5|=8m-20$ and $|E_6|=3m^2-16m+21.$

1. Now from the definition of multiplicative version of generalized first ZI, we have

$$MZ_1^k(M)=\prod_{xy\in E(M)}(d_x+d_y)^k$$
$$=\left[(d_x+d_y)^k\right]^{|E_1(M)|}\times\left[(d_x+d_y)^k\right]^{|E_2(M)|}$$
$$\times\left[(d_x+d_y)^k\right]^{|E_3(M)|}\times\left[(d_x+d_y)^k\right]^{|E_4(M)|}$$
$$\times\left[(d_x+d_y)^k\right]^{|E_5(M)|}\times\left[(d_x+d_y)^k\right]^{|E_6(M)|}$$
$$=(6)^{4k}\times(7)^{4k}\times(9)^{2k}\times(8)^{k(4m-10)}$$
$$\times(10)^{k(8m-20)}\times(12)^{k(3m^2-16m+21)}$$
$$=(12)^{3km^2}\times\left(\frac{5^8}{2^{12}\times 3^{16}}\right)^{km}\left(\frac{3^{29}\times 7^4}{2^4\times 5^{20}}\right)^k.$$

2. From the definition of multiplicative version of second generalized ZI, we have

$$\mathrm{MZ}_2^k(M)=\prod_{xy\in E(M)}\left(d_x\times d_y\right)^k$$

$$=\left[\left(d_x\times d_y\right)^k\right]^{|E_1(M)|}\times\left[\left(d_x\times d_y\right)^k\right]^{|E_2(M)|}$$

$$\times\left[\left(d_x\times d_y\right)^k\right]^{|E_3(M)|}\times\left[\left(d_x\times d_y\right)^k\right]^{|E_4(M)|}$$

$$\times\left[\left(d_x\times d_y\right)^k\right]^{|E_5(M)|}\times\left[\left(d_x\times d_y\right)^k\right]^{|E_6(M)|}$$

$$=(8)^{4k}\times(12)^{4k}\times(18)^{2k}\times(16)^{k(4m-10)}$$

$$\times(24)^{k(8m-20)}\times(36)^{k\left(3m^2-16m+21\right)}$$

$$=(36)^{3km^2}\times\left(\frac{2^8}{3^{24}}\right)^{km}\left(\frac{3^{30}}{2^{36}}\right)^k.$$

3. By the definition of multiplicative version of second generalized *GA* index, we have

$$GA^kII(M)=\prod_{xy\in E(M)}\left(\frac{2\sqrt{d_x\cdot d_y}}{d_x+d_y}\right)^k$$

$$=\left(\frac{2\sqrt{8}}{6}\right)^{4k}\times\left(\frac{2\sqrt{12}}{7}\right)^{4k}\times\left(\frac{2\sqrt{18}}{9}\right)^{2k}\times(1)^{k(4m-10)}$$

$$\times\left(\frac{2\sqrt{24}}{10}\right)^{k(8m-20)}\times(1)^{k(3m^2-16m+21)}$$

$$=\left[\frac{2^7\times3^{4m}\times5^{10}}{2^{4m}\times3^{10}\times7^4\times5^{8m}}\right]^k.$$

In theorem 1, if we take k=1 we get following results.

Corollary 2. Let M=H$_n$ be Hex board graph. Then multiplicative version of first ZI, multiplicative version of second ZI and multiplicative version of GA index of M are:

1. $MZ_1(M)=(12)^{3m^2}\times\left(\frac{5^8}{2^{12}\times3^{16}}\right)^m\left(\frac{3^{29}\times7^4}{2^4\times5^{20}}\right).$

2. $MZ_2(M)=(6)^{6m^2}\times\left(\frac{2^8}{3^{24}}\right)^m\left(\frac{3^{30}}{2^{36}}\right).$

3. $GAII(M)=\left[\frac{2^7\times3^{4m}\times5^{10}}{2^{4m}\times3^{10}\times7^4\times5^{8m}}\right].$

In theorem 1, if we take k=2 we get:

Corollary 3. Let M=H$_n$ be Hex board graph. Then the multiplicative versions of first and second harmonic indices of M are

1. $HII_1(M)=(12)^{6M^2}\times\left(\frac{5^{16}}{2^{24}\times3^{32}}\right)^m\left(\frac{3^{29}\times7^4}{2^4\times5^{20}}\right)^2,$

2. $HII_2(M)=(6)^{12m^2}\times\left(\frac{2^{16}}{3^{48}}\right)^m\left(\frac{3^{60}}{2^{72}}\right).$

In theorem 1, if we take $k=1/2$ we get:

Corollary 4. Let $M=H_n$ be Hex board graph. Then the multiplicative sum and product connectivity indices of M are

1. $SCII(M)=\left(\frac{1}{\sqrt{12}}\right)^{3m^2}\times\left(\frac{2^6\times3^8}{5^4}\right)^m\left(\frac{2^2\times5^{10}}{3^{29/2}\times7^2}\right),$

2. $PCII(M)=\left(\frac{1}{6}\right)^{3m^2}\times\left(\frac{3^{12}}{2^4}\right)^m\left(\frac{2^{18}}{3^{15}}\right).$

Theorem 5. Let $M=H_n$ be Hex board graph. Then the multiplicative version of ABC index of M is

$$ABCII(M)=\frac{5^{\frac{1}{2}(3m^2-16m+25)}}{2^{1/2(3n^2-4m+5)}\times3^{3m^2-14m+20}}.$$

Proof. By using definition and the edge partition given in theorem 1, we have

$$ABCII(M)=\prod_{xy\in E(M)}\sqrt{\frac{d_x+d_y-2}{d_x\cdot d_y}}$$

$$=\left(\sqrt{\frac{1}{2}}\right)^4\times\left(\sqrt{\frac{5}{12}}\right)^4\times\left(\sqrt{\frac{7}{18}}\right)^2\times\left(\sqrt{\frac{6}{16}}\right)^{4m-10}$$

$$\times\left(\sqrt{\frac{8}{24}}\right)^{8m-20}\times\left(\sqrt{\frac{10}{36}}\right)^{3m^2-16m+21}$$

$$=\frac{5^{\frac{1}{2}(3m^2-16m+25)}}{2^{1/2(3m^2-4m+5)}\times3^{3m^2-14m+20}}.$$

Multiplicative degree-depending TIs of L(Hn)

In this section, we will study $L(H_n)$. The following theorem is about the multiplicative versions of generalized ZI and GA index.

Theorem 6. Let L(M) be the line graph of Hex board H_n. Then the multiplicative versions of generalized first ZI, multiplicative version of generalized second ZI and generalized version of GA index of L(M) are:

1. $MZ_1^k(\mathrm{L(M)})=\left(2^2\times5\right)^{15km^2}\times\left(\frac{3^{68}\times7^{16}}{2^{104}\times5^{96}}\right)^{km}\left(\frac{2^{100}\times5^{166}\times17^6}{3^{204}\times7^{48}}\right)^k,$

2. $MZ_2^k(\mathrm{L(M)}) = (10)^{30km^2} \times \left(\frac{2^{56}\times 3^{24}}{5^{160}}\right)^{km} \left(\frac{5^{230}\times 7^{14}}{2^{218}\times 3^{60}}\right)^{k}$,

$GA^k\,II(\mathrm{L(M)}) = \left(2^{96}\times 3^{56}\times 5^{16}\times 7^{16}\right)^{mk}$

3. $\left[\frac{3^{174}\times 7^{53}}{2^{180}\times 5^{54}\times 11^{4}\times 13^{8}\times 17^{6}}\right]^{k}$.

Proof. Let *L(M)* be the line graph of Hex board graph. Then

$E_1(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 4, d_y = 4\}$,

$E_2(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 4, d_y = 6\}$,

$E_3(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 4, d_y = 8\}$,

$E_4(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 5, d_y = 5\}$,

$E_5(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 5, d_y = 6\}$,

$E_6(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 5, d_y = 7\}$,

$E_7(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 5, d_y = 8\}$,

$E_8(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 6, d_y = 6\}$,

$E_9(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 6, d_y = 8\}$,

$E_{10}(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 7, d_y = 8\}$,

$E_{11}(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 7, d_y = 10\}$,

$E_{12}(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 8, d_y = 8\}$,

$E_{13}(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 8, d_y = 10\}$,

$E_{14}(\mathrm{L(M)}) = \{xy \in E(\mathrm{L(M)}) | d_x = 10, d_y = 10\}$,

Such that

$|E_1| = 2,\ |E_2| = 8,$

$|E_3| = 4,\ |E_4| = 2,$

$|E_5| = 4,\ |E_6| = 4,$

$|E_7| = 8,\ |E_8| = 4m-12,$

$|E_9| = 16m-48,$

$|E_{10}| = 4, |E_{11}| = 6,$

$|E_{12}| = 8m-14,$

$|E_{13}| = 32m-100,$

$|E_{14}| = 15m^2-96m+152.$

1. Now from the definition of multiplicative generalized first ZI, we have

$$\begin{aligned}
\mathrm{MZ}_1^{\alpha}(G) &= \prod_{uv\in E(G)} (d_u + d_v)^{\alpha} \\
&= (8)^{2\alpha}\times(10)^{8\alpha}\times(12)^{4\alpha}\times(10)^{2\alpha}\times(11)^{4\alpha}\times(12)^{4\alpha}\times(13)^{8\alpha}\times(12)^{\alpha(4k-12)}\times(14)^{\alpha(16k-48)} \\
&\quad\times(15)^{4\alpha}\times(17)^{6\alpha}\times(16)^{\alpha(8k-14)}\times(18)^{\alpha(32k-100)}\times(20)^{\alpha(15k^2-96k+152)} \\
&= \left(2^2\times 5\right)^{15\alpha k^2}\times\left(\frac{3^{68}\times 7^{16}}{2^{104}\times 5^{96}}\right)^{\alpha k}\left(\frac{2^{100}\times 5^{166}\times 17^{6}}{3^{204}\times 7^{48}}\right)^{\alpha}.
\end{aligned}$$

2. Now from the definition of multiplicative version of generalized second ZI, we have

$$\begin{aligned}
\mathrm{MZ}_2^{\alpha}(G) &= \prod_{uv\in E(G)} (d_u \times d_v)^{\alpha} \\
&= (16)^{2\alpha}\times(24)^{8\alpha}\times(32)^{4\alpha}\times(25)^{2\alpha}\times(30)^{4\alpha}\times(35)^{4\alpha}\times(40)^{8\alpha}\times(36)^{\alpha(4k-12)}\times(48)^{\alpha(16k-48)} \\
&\quad\times(56)^{4\alpha}\times(70)^{6\alpha}\times(64)^{\alpha(8k-14)}\times(80)^{\alpha(32k-100)}\times(100)^{\alpha(15k^2-96k+152)} \\
&= (10)^{30\alpha k^2}\times\left(\frac{2^{56}\times 3^{24}}{5^{160}}\right)^{\alpha k}\left(\frac{5^{230}\times 7^{14}}{2^{218}\times 3^{60}}\right)^{\alpha}.
\end{aligned}$$

3. Now from the definition of multiplicative version of generalized geometric arithmetic index, we have

$$GA^k\,II(\mathrm{L(M)}) = \prod_{xy\in E(\mathrm{L(M)})}\left(\frac{2\sqrt{d_x\cdot d_y}}{d_x+d_y}\right)^{k}$$

$$=\left(\frac{2\sqrt{16}}{8}\right)^{2k}\times\left(\frac{2\sqrt{24}}{10}\right)^{8k}\times\left(\frac{2\sqrt{32}}{12}\right)^{2k}\times\left(\frac{2\sqrt{25}}{10}\right)^{2k}$$

$$\times\left(\frac{2\sqrt{30}}{11}\right)^{4k}\times\left(\frac{2\sqrt{35}}{12}\right)^{4k}\times\left(\frac{2\sqrt{40}}{13}\right)^{8k}$$

$$\times\left(\frac{2\sqrt{36}}{12}\right)^{k(4m-12)}\times\left(\frac{2\sqrt{48}}{14}\right)^{k(16m-48)}\times\left(\frac{2\sqrt{56}}{15}\right)^{4k}$$

$$\times\left(\frac{2\sqrt{70}}{17}\right)^{6k}\times\left(\frac{2\sqrt{64}}{16}\right)^{k(8m-14)}\times\left(\frac{2\sqrt{80}}{18}\right)^{k(32m-100)}$$

$$\times\left(\frac{2\sqrt{100}}{20}\right)^{k(15m^2-96m+152)}$$

$$=\left(2^{96}\times3^{56}\times5^{16}\times7^{16}\right)^{mk}\left[\frac{3^{174}\times7^{53}}{2^{180}\times5^{54}\times11^{4}\times13^{8}\times17^{6}}\right]^{k}.$$

In theorem 6, if we take *k=1* we get following results.

Corollary 7. Let L(M) be the line graph of Hex board H_n. Then multiplicative version of first ZI, multiplicative version of second ZI and multiplicative version of GA index of L(M) are:

1. $$MZ_1\left(\mathrm{L(M)}\right)=\left(2^2\times5\right)^{15m^2}\times\left(\frac{3^{68}\times7^{16}}{2^{104}\times5^{96}}\right)^{m}\left(\frac{2^{100}\times5^{166}\times17^{6}}{3^{204}\times7^{48}}\right),$$

2. $$MZ_2\left(\mathrm{L(M)}\right)=(10)^{30m^2}\times\left(\frac{2^{56}\times3^{24}}{5^{160}}\right)^{m}\left(\frac{5^{230}\times7^{14}}{2^{218}\times3^{60}}\right),$$

3. $$GAII\left(\mathrm{L(M)}\right)=\left(2^{96}\times3^{56}\times5^{16}\times7^{16}\right)^{m}\left[\frac{3^{174}\times7^{53}}{2^{180}\times5^{54}\times11^{4}\times13^{8}\times17^{6}}\right].$$

In theorem 6, if we take *k=2* we get following results.

Corollary 8. Let L(M) be the line graph of Hex board H_n. Then the multiplicative versions of first and second harmonic indices of L(M) are

1. $$HII_1\left(\mathrm{L(M)}\right)=\left(2^2\times5\right)^{30m^2}\times\left(\frac{3^{68}\times7^{16}}{2^{104}\times5^{96}}\right)^{2m}\left(\frac{2^{200}\times5^{332}\times17^{12}}{3^{408}\times7^{96}}\right),$$

2. $$HII_2\left(\mathrm{L(M)}\right)=(10)^{60m^2}\times\left(\frac{2^{112}\times3^{48}}{5^{320}}\right)^{m}\left(\frac{5^{460}\times7^{28}}{2^{436}\times3^{120}}\right).$$

In theorem 6, if we take $k=1/2$ we get following results.

Corollary 9. Let L(M) be the line graph of Hex board H_n. Then the multiplicative sum product connectivity indices of L(M) are

1. $$SCII\left(\mathrm{L(M)}\right)=\frac{1}{\left(2^2\times5\right)^{\frac{1}{2}m^2}}\times\left(\frac{2^{52}\times5^{48}}{3^{34}\times7^{8}}\right)^{m}\left(\frac{3^{102}\times7^{24}}{2^{50}\times5^{133}\times17^{3}}\right),$$

2. $$PCII\left(\mathrm{L(M)}\right)=\frac{1}{(10)^{15m^2}}\times\left(\frac{5^{80}}{2^{28}\times3^{12}}\right)^{m}\left(\frac{2^{109}\times3^{30}}{5^{115}\times7^{7}}\right).$$

Theorem 10. Let L(M) be the line graph of Hex board H_n. Then the multiplicative version of ABC index of L(M) is

$$ABCII\left(\mathrm{L(M)}\right)=\left(\frac{3^{15}}{2^{15/2}\times5^{15}}\right)^{m^2}\times\left(\frac{2^{14}\times5^{82}\times7^{4}}{3^{102}}\right)^{m}\left(\frac{2^{64}\times3^{166}\times11^{4}\times13^{2}}{5^{114}\times7^{14}}\right).$$

Proof. By using definition and the edge partition given in theorem 6, we have

$$ABCII\left(\mathrm{L(M)}\right)=\prod_{xy\in E(\mathrm{L(M)})}\sqrt{\frac{d_x+d_y-2}{d_x\cdot d_y}}$$

$$=\left(\sqrt{\frac{6}{16}}\right)^{2}\times\left(\sqrt{\frac{8}{24}}\right)^{8}\times\left(\sqrt{\frac{10}{32}}\right)^{4}\times\left(\sqrt{\frac{8}{25}}\right)^{2}\times\left(\sqrt{\frac{9}{30}}\right)^{4}$$

$$\times\left(\sqrt{\frac{10}{35}}\right)^{4}\times\left(\sqrt{\frac{11}{40}}\right)^{8}\times\left(\sqrt{\frac{10}{36}}\right)^{4k-12}\times\left(\sqrt{\frac{12}{48}}\right)^{16m-48}$$

$$\times\left(\sqrt{\frac{13}{56}}\right)^{4}\times\left(\sqrt{\frac{15}{70}}\right)^{6}\times\left(\sqrt{\frac{14}{64}}\right)^{8m-14}$$

$$\times\left(\sqrt{\frac{16}{80}}\right)^{32m-100}\left(\sqrt{\frac{18}{100}}\right)^{15m^2-96m+152}$$

$$=\left(\frac{3^{15}}{2^{15/2}\times5^{15}}\right)^{m^2}\times\left(\frac{2^{14}\times5^{82}\times7^{4}}{3^{102}}\right)^{m}\left(\frac{2^{64}\times3^{166}\times11^{4}\times13^{2}}{5^{114}\times7^{14}}\right).$$

Conclusion

Graphs are a very important tool, used to describe and represent algorithms, information structures, networks, social interaction, and flows of electricity, electricity and many more things. In this study, we delved into Hex board graph and line graph of hex board graphs. We computed multiplicative versions of several degree-based TIs. These TIs can help us to understand the chemical reactivity, biological activates and physical features. These results can also play an important part in the determination of the significance of Hex Board and line graph of Hex board graphs. For example, it has been experimentally proved that the first ZI is directly related with total π -electron energy. Also RI is useful for determining physio-chemical properties of alkanes as noticed by the chemist Melan Randić in 1975. He noticed the correlation between the RI and several physico–chemical properties of alkanes like, enthalpies of formation, boiling points, chromatographic retention times, and vapor pressure and surface areas.

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